Propagation of cracks through an ice shelf as precondition for calving: numerical experiments with an idealised glacial system

Arno C. Hammann[1] and Henner Sandhäger[2]

[1]*Internship at* [2]
[2]*Alfred-Wegener-Institut für Polar- und Meeresforschung, Postfach 120161, D-27515 Bremerhaven*

Abstract

We investigated mechanisms which might lead to the calving of large icebergs from floating ice shelves in Antarctica. To this end, a number of experiments were carried out with a numerical model of ice shelf dynamics. The results show that it is possible to distinguish between favourable and unfavourable regions for crack growth. We also show that it is possible to delineate crack propagation paths on the basis of principal stress trajectories. The scope of this study is limited by the inability to perform time-dependent simulations of ice shelves containing cracks with the present numerical model.

Introduction

Mechanisms have been proposed to explain the calving of small icebergs (with width about equal to one ice thickness) from the ice fronts of tidewater and lake-bounded glaciers (Reeh, 1968; Fastook and Schmidt, 1982). However, no coherent theory exists which is concerned with the calving of large tabular icebergs from Antarctic ice shelves (with main axis lengths of tens to hundreds of kilometres). Such calving events appear to follow a certain basic pattern: inlets open more or less perpendicular to the ice shelf front, intermittently widening and growing, until the final break-off of icebergs occurs rapidly, and frequently, at an angle with the original orientation of the inlet (sometimes joining two inlets).

While research is under way to investigate the fracture mechanical properties of Antarctic shelf ice (e.g., Rist et al., 2002), the question of climatic sensitivity of ice shelves is becoming ever more relevant. This issue can hardly be addressed in numerical simulation studies without the explicit consideration of calving processes.

Therefore, this study aims at formulating some first conceptual ideas concerned with ice shelf calving mechanisms. We focus on the question whether there are forces *internal* to the ice shelf system which can drive such calving mechanisms. Internal stress fields of idealised ice shelf configurations are investigated here by means of a numerical model of ice shelf dynamics (itself part of a coupled grounded ice sheet – ice shelf model). In particular, we studied the effects of inlets penetrating into the ice shelf.

As a result, we demonstrate how a numerical model can be used to assess, in principle, whether fractures will propagate in different areas of an ice shelf. Quantitative predictions are not possible without further research, but it appears that established criteria for material strength (e.g., the von Mises yield criterion) can be adapted easily to our conceptual framework. We also show how the paths of fractures propagating through the ice shelf can be determined.

Description of numerical experiments

The numerical model of ice shelf dynamics which we used in this study is part of a coupled ice sheet – ice shelf model. An initial diagnostic (time-independent) version of this model was developed by Sandhäger (2000) and later extended to enable time-dependent modelling (as described in Bleker, 2002).

The model is based on differential equations which describe material properties and continuum mechanics of the system. The so-called shallow shelf approximation (*SSA*, e.g. Weis et al., 1999) is used to simplify calculations. SSA ignores vertical shear within an ice shelf body and thus treats velocities of ice flow as depth-invariant. The equations are solved numerically on a regular (5×5 km) grid. In diagnostic mode, the model takes as main input the geometrical layout of the glacial system and mean annual temperatures and accumulation rates at the ice shelf surface, from which it calculates ice flow velocities. In time-dependent mode, it requires additional climatic input parameters which govern the evolution of the ice shelf geometry.

In this report, we describe a series of experiments through which we attempted to simulate the reaction of stress and velocity fields in the ice shelf body to the propagation of fractures within the ice. We restrict ourselves to an idealised ice shelf geometry in (two-dimensional) plan view, the same as has been used by Grosfeld et al. (this issue), who coupled the present ice shelf model to an ocean circulation model. We consider stresses in the uppermost layer of the ice shelf body only, since this top layer represents in our simplified model approach the coldest and stiffest part of the ice shelf, and therefore has been argued to constrain decisively ice shelf dynamics.

'Cracks' or 'inlets' are simulated in a very simple and crude way: grid cells identified as containing 'ice' are changed to containing 'ocean' along a line corresponding to the crack. In this way, we can only simulate features substantially larger than the 5-km grid resolution. Also, at present, we cannot use the model in time-dependent mode once these inlets are introduced, since the opening and migration of cracks represent cases of grid cell behaviour which are not yet considered in the model rules. The one simple rule we use to determine the path of crack propagation is that fractures open perpendicular to the direction of greatest tensile stress (according to the '$\sigma(\theta)_{max}$' theory; cf. Ingraffea, 1987). Inlets are introduced at more or less arbitrary locations and then extended incrementally; that is, only five grid cells are 'cracked' at a time, and stress patterns are re-calculated after each step. The reason for this is that we did not know from the outset whether the way the crack modifies the stress field during propagation would significantly change its eventual path.

Results

The stress fields which characterise the different ice shelf geometries of our experiments are presented here in terms of principal and effective deviatoric stresses. Although the absolute stresses dominate the fracture processes, the use of deviatoric stresses (absolute stresses from which the hydrostatic stress has been subtracted) should not pose a problem since both quantities are proportional. Principal stresses are obtained by rotating, at any point, the coordinate axes such that shear stresses disappear. In two dimensions, one obtains the values and directions of two principal stresses (at right angles to each other), where the first principal stress is defined as the numerically larger, the second as the smaller one. Across the entire ice shelf, the first principal stress is tensile (positive), and thus indicates the direction perpendicular to which fractures are expected to propagate in most cases (cf. Ingraffea, 1987). The effective stress is defined as the second invariant of the stress tensor and reflects the overall state of stress at any point (cf. Paterson, 1994).

Stress fields from an experiment where one crack is introduced at one side of the ice front show pronounced anomalies which migrate ahead of the propagating crack tip. Particularly remarkable are the nearly circular anomalies in the distribution of the effective stress (Fig. 1). The second principal stress shows a pronounced tendency to produce 'arches' of extreme values (Fig. 3), which have already been noticed and connected to a stability criterion for ice shelves by Doake et al. (1998).

Experiments with several cracks introduced in different areas in the vicinity of the ice shelf front resulted in pronounced stress anomalies for some, but not all, of the cracks (Fig. 4). An 'arch' of high second principal stress at times connects two inlets (Fig. 5). Although the pattern of stress trajectories (indicating orientations of principal stresses) changes while a crack is propagating incrementally, the eventual path does seem predetermined by the pre-cracking configuration (Fig. 6).

Discussion

The stress anomalies resulting from our experiments compare favourably with theoretical predictions. Figure 2 shows the pattern of effective stresses around a crack tip calculated from equations given by fracture mechanics theory. Fracture mechanics describes the effects of pre-existing cracks within a given stress field. The basic idea is that the crack serves as a 'concentrator' which focuses stresses at the crack tips (Atkinson, 1987). The simplest relations are obtained for the idealised case of elastic-brittle material behaviour; this variant of fracture mechanics is commonly referred to as *linear-elastic fracture mechanics* (LEFM).

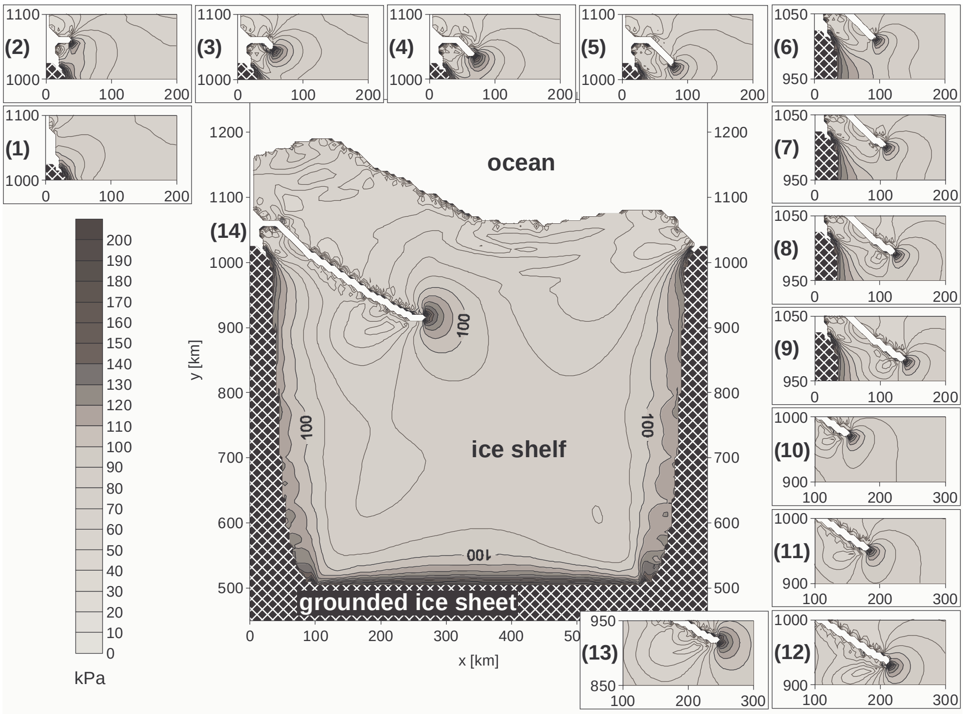

Figure 1: Distribution of effective deviatoric stresses in the vicinity of a propagating crack tip. The crack was broken into the ice shelf in increments of five cells at a time, but the model was only used in diagnostic mode (without consideration of time).

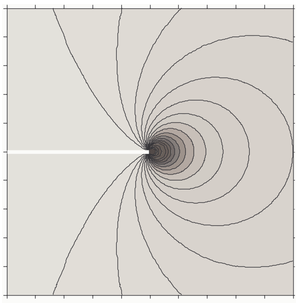

Figure 2: Distribution of the effective stresses calculated for a crack propagating into an ideal linear-elastic solid. An arbitrary value for the remotely applied stress is chosen; therefore, no scale is given.

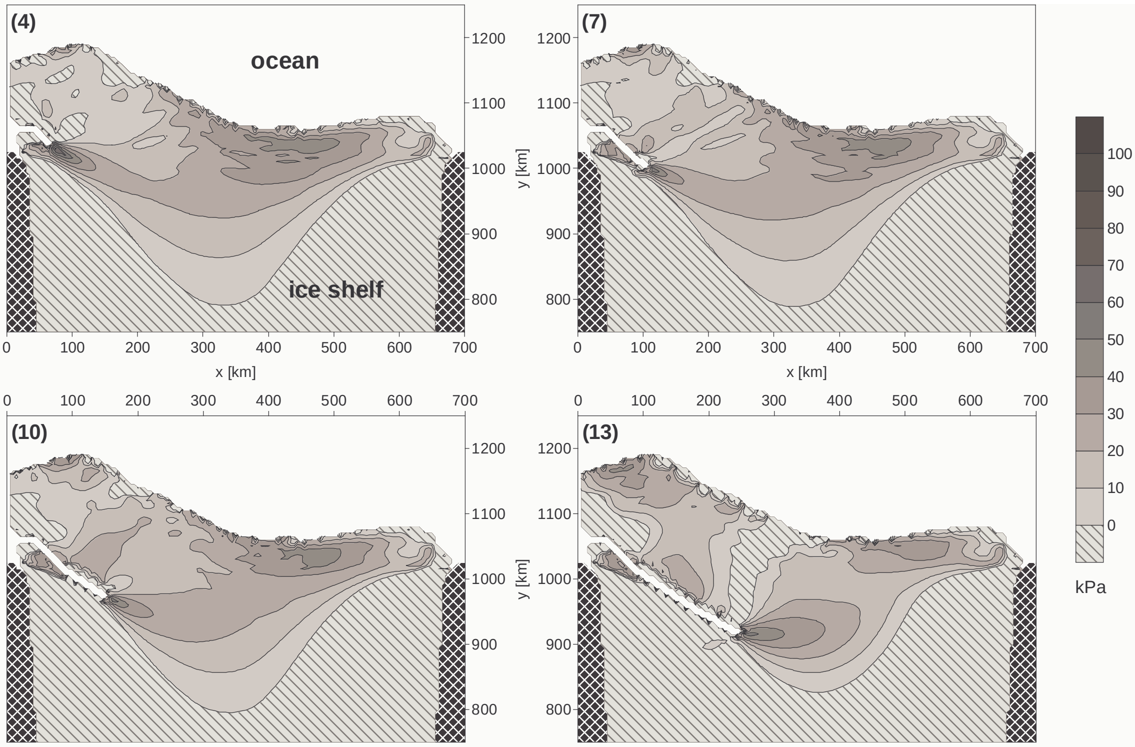

Figure 3: Distribution of positive (tensile) second principal deviatoric stresses in the area of the propagating crack.

In LEFM, the stresses at the crack tip are associated with a so-called *stress intensity factor*, K, which can, in two dimensions, be calculated from (Atkinson, 1987)

$$K = Y \sigma_r (\pi c)^{1/2} \quad (1)$$

where σ_r is the remote applied stress and c the crack half-length. Y is a correction factor to account for crack and stress field geometries; it is, however, often set to 1 (corresponding to a straight crack in a field of uniform stress). An existing crack will grow larger if K exceeds a critical value K_c, but will stop to grow if $K < K_c$. The direction of crack growth is given slightly differently by two competing theories, the already mentioned $\sigma(\theta)_{max}$ theory (propagation perpendicular to direction of greatest tensile stress), and the $S(\theta)_{min}$ theory, which predicts crack growth in the direction of minimum strain energy density (which is more difficult to apply in practice; cf. Ingraffea, 1987).

Nonlinear effects can be considered by assuming a 'process zone' at the crack tip, where the material does not behave in a linear-elastic way. For this zone, appropriate constitutive models can be formulated which describe the deformation processes necessary to induce crack opening (see Ingraffea (1987) for a well-illustrated example). However, LEFM applies even if the process zone is small compared to crack length or ligament (the distance between crack tip and the next open surface). Also, whereas K_c, as a material property, is considered constant in the LEFM case, it may depend on crack length with materials that behave nonlinearly (e.g., if strain hardening or softening occurs).

Our numerical model reproduces the patterns predicted by LEFM remarkably well (Fig. 2), despite the theory not being implemented explicitly in the model. But as long as basic material properties are considered by a numerical model, it should be capable of reproducing the predictions of LEFM independently. In fact, it should even be capable of reproducing nonlinear material behaviour.

One of the inferences that can be drawn from basic LEFM is that cracks will continue to grow once a critical initial crack length is exceeded. Also, the dependence of K on crack length means that crack propagation is possible even under very low stresses, similar to those found in ice shelves and glaciers (in the order of hundreds of kPa at the highest). Alternative approaches to fracture of materials (mainly so-called strength-of-materials approaches) predict failure often only at much higher stresses. This is one of the reasons why fracture mechanic approaches were adopted in glaciology in the first place (e.g., Rist et al. , 1996, 1999, 2002).

Ice is not an elastic-brittle solid. However, using our numerical model, we may be able to make predictions without deciding whether LEFM is applicable or not. Figure 1 shows a propagating crack which continues to re-create the circular anomaly of the effective stress field at its tip. Possibly, the behaviour of ice can be modelled satisfyingly by assuming a process zone with an associated constitutive model for crack opening based on effective stress. In fact, such models exist (e.g., the so-called *von Mises yield criterion*) and have been proposed for use in connection with process zones (Ingraffea, 1987).

Figure 4, on the other hand, shows that some cracks do not exhibit these pronounced anomalies. It may thus be possible to decide on the basis of such 'test cracks' whether cracks or inlets are likely to grow or will more probably come to a rest. In addition, it appears that the direction of crack propagation can be decided on the basis of the pre-crack configuration of principal stress trajectories (Fig. 6).

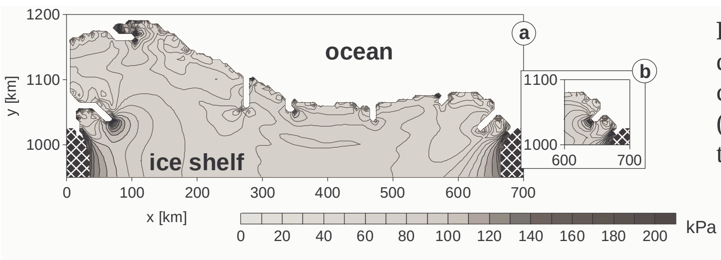

Figure 4: Distribution of the effective deviatoric stresses (**a**) for a model configuration containing seven inlets and (**b**) for the crack at the right-hand edge of the ice shelf, but for a shorter crack length.

These results should be seen in the light of two limitations of our experiments. First, we have not yet been able to perform time-dependent model runs. This leads to unrealistic velocity and stress fields if the crack grows too far toward the ice shelf interior. The widening and migration of an inlet is controlled by dynamic processes, but their implementation requires our present numerical model to be modified. As a result, we cannot simulate yet the often slow and intermittent process of inlet growth (since there is no temporal dimension implied in our experiments), and the final break-off of large icebergs. This break-off occurs often rapidly and along fractures which open in directions different from the original inlet orientation (possibly, pre-existing rupture zones exert significant influences here).

Secondly, the existence of anomalies at crack tips depends in our model crucially on the exact formulation of a boundary condition for the ice front (cf. Mayer, 1996). At present, we use a rough parameterisation to calculate the decisive freeboard heights at the ice shelf front. But in order to validate our results for the stress fields, it will be necessary to investigate how this boundary condition is best formulated or parameterised in order to achieve the most realistic results.

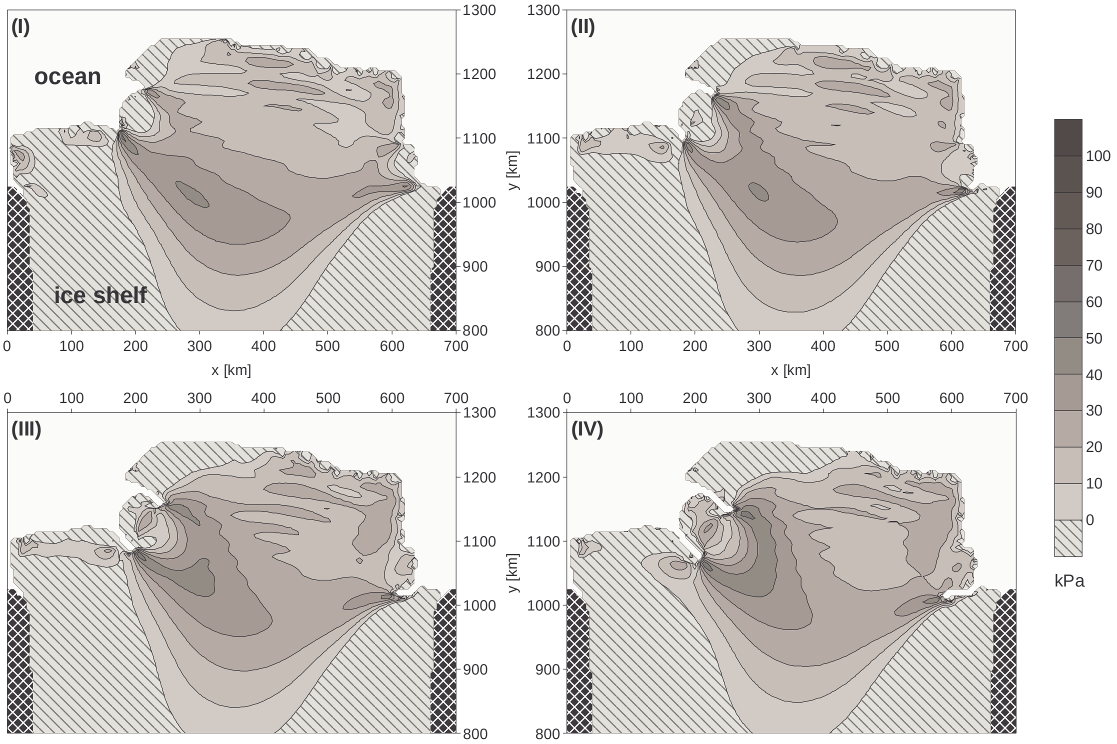

Figure 5: Distributions of positive (tensile) second principal deviatoric stresses for a model simulation where three cracks were forced to propagate into the ice shelf. Again, no temporal dimension is implied.

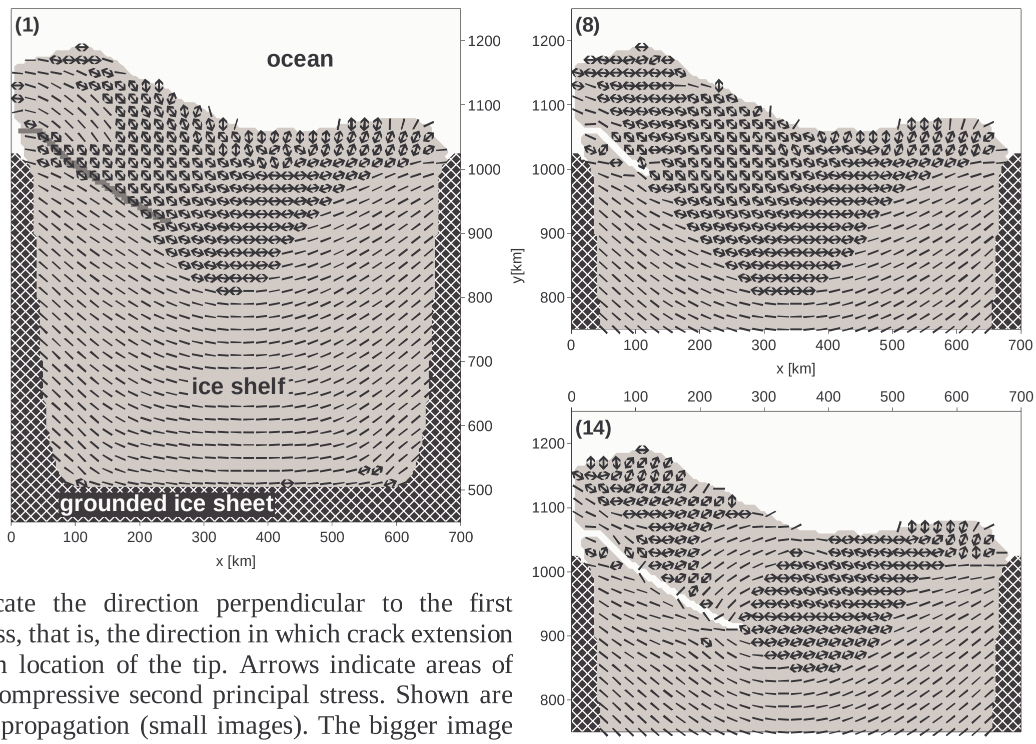

Figure 6:
Trajectories here indicate the direction perpendicular to the first principal deviatoric stress, that is, the direction in which crack extension will occur from a given location of the tip. Arrows indicate areas of tensile, lines areas of compressive second principal stress. Shown are two steps during crack propagation (small images). The bigger image shows the trajectories corresponding to the initial model configuration (without crack), but with the subsequent path of the crack overlain.

Conclusions and Outlook

We believe that numerical models of ice shelf dynamics can be used to answer, at least, the following two questions concerning large-scale iceberg calving:

(1) In what regions of an ice shelf pre-existing flaws and cracks are likely to grow further?

(2) In which direction will these cracks propagate? (They will propagate perpendicular to the orientation of the trajectories of the first principal stress.)

It will be necessary, as the next step towards improving our understanding of the processes of iceberg calving, to modify our existing numerical model of ice shelf dynamics such that the effects of crack widening and migration are considered. This includes the development of a better representation of cracks within the model domain.

In the meantime, there remains the question whether anomalies in the stress fields, in particular in the field of the second principal stress with its conspicuous ‘arches’, can be used to predict the location of post-calving ice fronts (compare Doake et al., 1998; Doake, 1996). Maybe, these arches correspond to zones which are more vulnerable to the influence of internal stresses and external forces (e.g., tidal flexure), and thus determine potential fracture paths.

References

Atkinson, B.K. (1987), Introduction to fracture mechanics, in: Atkinson, B.K., *Fracture mechanics of rock*, 1–23. London: Acad. Pr.

Bleker, K. (2002), *Numerische Experimente zum Einfluß von Felsbettneigung und Klimarandbedingungen auf Inlandeis-Schelfeis-Systeme mit aktiver Übergangszone*. Unpublished thesis, Westfälische Wilhelms-Universität Münster, Germany.

Doake, C.S.M. (1996), Strain-rate trajectories and isotropic points on Filchner Ronne Ice Shelf, Antarctica, in: Oerter, H. (ed.): *Filchner-Ronne Ice Shelf Programme Report* **10**, 16–22. Alfred Wegener Institute for Polar and Marine Research, Bremerhaven.

Doake, C.S.M., H.F.J. Corr, H. Rott, P. Skvarca and N.W. Youngs (1998), Breakup and conditions for stability of the northern Larsen Ice Shelf, Antarctica, *Nature* **391**, 778–780.

Fastook, J.L. and W.F. Schmidt (1982), Finite element analysis of calving from ice fronts, *Ann. Glaciol.* **3**, 103–106.

Grosfeld, K., H. Sandhäger and M.A. Lange (2003), Sensitivity of a coupled ice shelf - ocean system to changed oceanographic boundary conditions, in: Oerter, H. and L.H. Smedsrud (eds.): *Filchner-Ronne Ice Shelf Programme Report* **14**. Alfred Wegener Institute for Polar and Marine Research, Bremerhaven.

Ingraffea, A.R. (1987), Crack initiation and propagation, in: Atkinson, B.K., *Fracture mechanics of rock*, 71–110. London: Acad. Pr.

Mayer, C. (1996), Numerische Modellierung der Übergangszone zwischen Eisschild und Schelfeis, *Ber. Polarf.* **214**, Alfred Wegener Institute for Polar and Marine Research, Bremerhaven.

Paterson, W.S.B. (1994), *The physics of glaciers*. Elsevier Science Ltd., Oxford, UK.

Reeh, N. (1968), On the calving of ice from floating glaciers and ice shelves, *J. Glaciol.* **7**, 215–232.

Rist, M.A., P.R. Sammonds, S.A.F. Murrell, P.G. Meredith, H. Oerter and C.S.M. Doake (1996), Experimental fracture and mechanical properties of Antarctic ice: preliminary results, *Ann. Glaciol.* **23**, 284–292.

Rist, M.A., P.R. Sammonds, S.A.F. Murrell, P.G. Meredith, C.S.M. Doake, H. Oerter and K. Matsuki (1999), Experimental and theoretical fracture mechanics applied to Antarctic ice fracture and surface crevassing, *J. Geophys. Res.* **104/B2**, 2973–2987.

Rist, M.A., P.R. Sammonds, H. Oerter and C.S.M. Doake (2002), Fracture of Antarctic shelf ice, *J. Geophys. Res.* **107/B1**, 10.1029/2000JB000058.

Sandhäger, H. (2000), *Quantifizierung eisdynamischer und massenhaushaltsrelevanter Basisgrößen eines antarktischen Inlandeis-Schelfeis-Systems unter Einsatz eines numerischen Fließmodells*. PhD-thesis, Westfälische Wilhelms-Universität Münster, Germany.

Weis, M., R. Greve and K. Hutter (1999), Theory of shallow ice shelves, *Continuum Mech. Thermodyn.* **11**, 15–50.