Das Problem der Wettervorhersage, betrachtet vom Standpunkte der Mechanic und der Physik
V. Bjerknes

Meteorolgische Zeitschrift, Januar 1904, pp. 1-7.

English Translation by Y. Mintz, Los Angeles, 1954

WEATHER FORECASTING AS A PROBLEM IN MECHANICS AND PHYSICS

by V. Bjerknes

If it is true, as every scientist believes, that subsequent atmospheric states develop from the preceding ones according to physical law, then it is apparent that the necessary and sufficient conditions for the rational solution of forecasting problems are the following:

1. A sufficiently accurate knowledge of the state of the atmosphere at the initial time.
2. A sufficiently accurate knowledge of the state of the laws according to which one state of the atmosphere develops from the other.

I.

The determination of the state of the atmosphere at the initial time is the task of observational meteorology. This problem has not yet been solved to the extent that is necessary for rational forecasting. There are two major gaps in the observations. The first is that only land stations participate in the daily programs of the weather services. Over the seas, which cover four-fifths of the earth's surface and must therefore exert an overwhelming influence, no observations are made for the purposes of current weather analysis. Furthermore, the observations which are used in current analysis are only made at the surface of the earth and all data pertaining to the state of the higher layers of the atmosphere are missing. But we already have the technical means, which will enable us to fill these two gaps. With the help of wireless telegraphy we will be able to include among the reporting stations the ships moving in fixed routes. And to judge by the great forward steps, which have been made in recent years in the techniques of upper air soundings, it will be possible to obtain daily observations of the higher atmospheric layers not only from fixed land positions but also from traveling stations on the sea. We can hope, therefore, that the time will soon come when either as a daily routine, or for certain designated days, a complete diagnosis of the state of the atmosphere will be available. The first condition for putting forecasting on a rational basis will then be satisfied.

II.

The second problem then arises as to whether we know, with sufficient accuracy the laws according to which one state of the atmosphere develops out of another. The atmospheric processes are of a mixed mechanical and physical nature. Each one of these processes can be expressed in one or more mathematical equations according to mechanical or physical principles. We have sufficient knowledge of the laws according to which the atmosphere develops if we can set up as many independent equations as there are unknown quantities. From a meteorological point of view, the state of the atmosphere is specified, at an arbitrary time, if we can determine for that time at each point the velocity, density, pressure, temperature, and humidity of the air. The velocity, as a vector, is given by three scalar quantities, the three velocity components, and one must therefore deal with seven unknown quantities.

To compute these quantities, we can set up the following equations:

1. The three hydrodynamical equations of motion. These are differential relations between the three velocity components, the density and pressure.
2. The continuity equation, which expresses the principle of the conservation of mass during motion. This equation is again a differential relation between the velocity components and the density.
3. The equation of state of atmospheric air, which is a relation in finite form between the density, pressure, temperature, and humidity of a given mass of air.
4. The two laws of thermodynamics, which allow us to set up two differential relations giving the rates of change of energy and entropy during the changes of state which are taking place.

These equations introduce no new unknowns into the problem, as the energy and entropy are expressed by the same variables, which appear in the equation of state and connect the changes of these quantities with other quantities considered as known. These other quantities are, first, the work done by the mass of air, which is determined by the same variables that appear in the dynamical equations; and secondly, the amount of heat given up or received by the mass of air, which is determined by the physical data on incoming and outgoing radiation and on conduction where the air is in contact with the ground.

It should be emphasized that a basic simplification of the problem can be achieved if there is no condensation or evaporation of water, so that the water vapor of the air can be considered as a constant constituent. Then the problem will have one variable less, and one of the equations, the one that comes from the second law of thermodynamics, can be eliminated. On the other hand, if we had to deal with several variable constituents of the atmosphere, then the second law of thermodynamics would give a new equation for each new constituent. For the computation of the normally occurring seven variables, we can set up seven independent equations. So that, as we now see the problem, we must conclude that we do have sufficient knowledge of the laws of atmospheric processes upon which a rational weather forecasting system can be based.

But it must be admitted that we could have overlooked important factors on account of the incomplete ness of our knowledge. The interference of cosmic effects of an unknown kind may be imagined. Further more, the large-scale atmospheric phenomena are accompanied by a long train of subsidiary effects, for example those of an electrical or optical nature, and the question is to what extent such subsidiary effects could react in a significant way on the course of the atmospheric processes. These reactions exist, of course: for instance, the rainbow modifies the distribution of incoming radiant energy from the sun, and electric potentials influence the condensation processes. But until now there is no evidence that processes of this kind react upon the large-scale atmospheric processes in any significant way. Yet in any case, the scientific method is to start with the simplest problem that can be formulated, which is the problem posed above, of seven variables and seven equations.

III.

Of the seven equations, only one, the equation of state has a finite form. The other six are partial differential equations. Of the seven unknowns, one can be eliminated with the aid of the equation of state, and the problem then becomes the integration of a system of six partial differential equations with six unknowns, and with the utilization of initial conditions as given by the observations of the initial state of the atmosphere.

An exact analytical integration of the system of equations is out of the question. Even the computation of the motion of three mass-points, which influence each other according to a law as simple as that of Newton, exceeds the limits of today's mathematical analysis. Naturally there is no hope of understanding the motion of all the points of the atmosphere, which have far more complicated reactions upon one another. Moreover, the exact analytical solution, even if we could write it down, would not give the result, which we need. For to be practical and useful, the solution has to have a readily seen, synoptic form and has to omit the countless details which would appear in every exact solution. The prognosis need only deal, therefore, with averages over sizeable distances and time intervals; for example, from degree of meridian to degree of meridian and from hour to hour, but not from millimeter to millimeter or second to second.

We therefore forego any thought of analytical methods of integration and instead, pose the problem of weather prediction in the following practical form: Based upon the observations that have been made, the initial state of the atmosphere is represented by a number of charts which give the distribution of the seven variables from level to level in the atmosphere. With these charts as the starting point, new charts of a similar kind are to be drawn which represent the new state from hour to hour. For the solution of the problem in this form, graphical or mixed graphical and numerical methods are appropriate, which methods must be derived either from the partial differential equations or from the dynamical-physical principles, which are the basis of these equations. There is no reason to doubt, beforehand, that these methods can be worked out. Everything will depend upon whether we can successfully divide, in a suitable way, the total problem of insurmountable difficulty into a number of partial problems of which none is too difficult.

IV.

To accomplish this division into partial problems, we have to draw upon the general principle, which is the basis of the infinitesimal calculus of several variables. For purposes of computation, one can replace the simultaneous variation of several variables with the sequential variation of single variables or groups of variables. If one goes to the limit of infinitesimal intervals, one arrives at the approximation methods of finite difference computations and mechanical quadrature, which we must use here.

These principles cannot be used blindly, however, because the practical usefulness of the method will depend on the natural grouping of the variables so that one gets comprehensible partial problems, well defined in mathematical and physical respects. Above all, the first division will be basic. It must follow a natural line of division in the overall problem. One such natural line of division may be as indicated. It follows the boundary-line between the specifically dynamic and specifically physical processes out of which the atmospheric processes are composed. The division along this boundary-line divides the overall problem into pure hydrodynamic and thermodynamic partial problems.

The link which ties the hydrodynamical and the thermodynamical problems together is very easy to cut, so easy indeed that the theoretical hydrodynamicists have fully used it to avoid every serious contact with meteorology; for the connecting link is the equation of state. If we suppose that temperature and moisture do not enter into this equation, then we come to a "supplementary" equation, used ordinarily by the hydrodynamicists, which is a relation only between density and pressure. Thereby one is led to the study of fluid motions under such circumstances that each explicit consideration of the thermodynamic processes automatically falls away.

Instead of making the temperature and the moisture disappear entirely from the equation of state, we can regard them, for short time intervals, as given quantities, with values derived from the observations or from the preceding calculations. When the dynamical problem for the time interval is solved, then one computes afterwards new values of temperature and moisture according to purely thermodynamical methods. These one regards as given quantities when one solves the hydrodynamical problem for the next time interval, and so on.

V.

This, then is the general principle for the first subdivision of the main problem. In the practical solution of this problem there are several different ways in which the separation may be done, according to the manner in which one introduces the hypotheses about temperature and moisture. But there is no need to go into more detail in a general discussion of this kind.

The next major question will be, however, whether the hydrodynamic and the thermodynamic partial problems can be individually solved in a sufficiently simple way. We consider, first, the hydrodynamic problem, which is the principal one; because the dynamic equations are the true prognostic equations. Only through them is time introduced as an independent variable in the problem. The thermodynamic equations do not contain time. The hydrodynamic problem lends itself well to graphical solutions. Instead of computing with the three dynamical equations, one can execute simple parallelogram constructions for a suitable number of selected points, with graphical or visual interpretation for the regions in between. The main difficulty will lie in the restriction on the motion, which follows from the equation of continuity and the boundary conditions. But the test of whether the continuity equation is satisfied or not can also be performed by graphical methods, and in doing so one can take into account the topography of the earth, performing the construction on charts which represent this topography in the usual way.

One will not encounter great mathematical difficulties in the solution of the hydrodynamical partial problems. However, there is a serious gap in our knowledge of the factors, which we must take into account, as we have a very incomplete knowledge of the frictional stress in the atmosphere. True friction depends upon velocity differences in the infinitesimally small, but meteorologists are forced to deal with the average movements of large masses of air. One cannot, therefore, apply the frictional terms of the hydrodynamical equations by using the coefficients of friction found in laboratory experiments, but one must draw upon empirical results about the effective resistance opposing the motion of large masses of air. However, we already have sufficient data of this kind to make the first attempts in the computational prediction of air movements and these attempts will create, in time, the necessary corrections and completions.

The thermodynamical partial problem can be considered to be much simpler, in mathematical respect, than the hydrodynamical. From the solution to the hydrodynamical problem one obtains the work done by the air masses during their displacement. Knowing this work, and knowing the amounts of heat introduced during the time interval by incoming radiation and given up by outgoing radiation, one computes the new distribution of temperature and moisture according to known thermodynamical principles. These computations will not be more difficult in mathematical respects than similar computations in laboratory experiments where masses of air are at rest in a closed space. We have extensive pioneering work also in the investigations of Hertz, v. Bezold, and others.

As in the hydrodynamical problem, the main difficulty will be the lack of knowledge of the different factors with which the computations are to be carried out. Estimates of the amount of heat, which the air masses receive as the difference between incoming and outgoing radiation, and estimates of the amount of water which evaporates from the surface of the ocean or which condenses into clouds and falls as rain, will be very uncertain in the beginning. However, we have sufficient knowledge for a trial performance of the first computation, and through continued work will gradually find more exact values of the constants, which relate to the different countries and oceans, to different heights in the atmosphere, to different weather situations, to different amounts of cloudiness, and so forth.

VI.

It is certain that we will not encounter insurmountable mathematical difficulties in following through with these methods. After the graphical techniques have been worked out and the necessary tabular aids have been assembled, the individual operations will probably be easy to execute. The number of individual operations does not have to be excessively large. The number will depend on the length of time interval for which the dynamic partial differential problem is to be solved. The shorter one chooses the time interval, the more involved the work becomes, but so does the result become more exact; the longer one chooses the time interval, the faster one arrives at the goal, but at the cost of accuracy. A final decision about the best time interval to use can be determined only by experience. Even if striving for accuracy, a one-hour time interval should suffice. For air masses will only exceptionally travel longer distances in an hour than a degree of longitude, and only exceptionally will their path appreciably change curva ture during that time. Thereby the conditions are fulfilled under which we can carry out simple parallelogram constructions with straight-line sections. If one gains sufficient experience and learns to utilize instinct and visual estimates, one would probably be able to work easily with much larger time intervals, such as six hours. For a 24-hour weather forecast, one would then carry out the hydrodynamic construction four times, and four times compute the thermodynamic corrections of temperature and moisture.

It may be possible some day, perhaps, to utilize a method of this kind as the basis for a daily practical weather service. But however that may be, the fundamental scientific study of atmospheric processes sooner or later has to follow a method based upon the laws of mechanics and physics. And thereby we will arrive, necessarily, at a method of the kind outlined here. If this is admitted, the general plan for dynamical-meteorological research is given.

The main task of observational meteorology will be to obtain regular simultaneous observations of all parts of the atmosphere, at the surface of the earth and aloft, over land and over sea. The first task of the theoretical meteorologist will be to work out, on the basis of these observations, the best possible overall picture of the physical and dynamical state of the atmosphere at the time of these observations. And this representation must have such a form that will enable it to serve as the starting point for weather prediction by rational dynamical-physical methods.

Even the first preliminary task is a sizeable one. For it is of course much more difficult to represent the state of the atmosphere at all elevations than only at sea level, as it is now done. In addition, our direct observations of the higher layers of the air will always be very limited. One must therefore use each observation from the higher levels to the utmost. From the directly observable quantities, one has to compute to the greatest extent all accessible data about the non-observable ones. In doing this, one has to utilize the physical relationships between the quantities. Even to construct a coherent picture of the total state of the atmosphere out of scattered observations, one has to use to a large extent, dynamical physical methods.

The second, and most important task of theoretical meteorology will finally be to construct, with this representation of the state of the atmosphere as the starting point, the representation of the future states, either according to the methods outlined here, or by methods of a similar kind. The comparison of the predicted fields with those which are given afterwards by the observations will reveal the general accuracy of the method, and at the same time will provide empirical knowledge of better values of the constants, as well as hints on the improvement of the method.

On later occasions I shall return to the various principal points of this program.